PRE BOARD PAPER SESSION 2018-19

CLASS XII CODE 18 SET-1

Subject: Mathematics (Date : 25th JAN. 2019)

Time allowed: 3 hours **Maximum Marks : 100**

General Instructions:
(i) All questions are compulsory.
(ii) This question paper contains 29 questions.
(iii) Question 1- 4 in Section A are very short-answer type questions carrying 1 mark each.
(iv) Question 5-12 in Section B are short-answer type questions carrying 2 marks each.
(v) Question 13-23 in Section C are long-answer-I type questions carrying 4 marks each. (vi) Question 24-29 in Section D are long-answer-II type questions carrying 6 marks each.
(vii)There is no overall choice. However, internal choice has been provided in 01 question of one mark each and 03 questions of 2 marks each 03 questions of four marks each and 03 questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.
(viii) Use of calculators in not permitted. You may ask for logarithmic tables, if required.

SECTION – A

1. If A and B are two matices such that AB =A and BA = B then what is value of B^2.
2. Find the position vector of a point R which divides the line joining two points $\mathbf{P}(\hat{\mathbf{i}}+2\hat{\mathbf{j}}-\hat{\mathbf{k}})$ and $\mathbf{Q}(-\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}})$ in the ratio 2 : 1 Internally

Or

Calculate the modulus along the along the sum of the vectors $2\hat{\mathbf{i}}+\hat{\mathbf{j}}+4\hat{\mathbf{k}}$, $3\hat{\mathbf{i}}-2\hat{\mathbf{j}}+7\hat{\mathbf{k}}$, $5\hat{\mathbf{i}}+2\hat{\mathbf{j}}-3\hat{\mathbf{k}}$

3. Let $\vec{a}=5\hat{i}-\hat{j}+7\hat{k}$ and $\vec{b}=\hat{i}-\hat{j}+m\hat{k}$. Find m, such that $\vec{\mathbf{a}}+\vec{\mathbf{b}}$ and $\vec{\mathbf{a}}-\vec{\mathbf{b}}$ are orthogonal.
4. Find the co-ordinates of the point, where the line through (5, 1, 6) and (3, 4, 1) crosses the yz-plane.

SECTION -B

5. Prove that $3\cos^{-1}x=\cos^{-1}(4x^3-3x)$ if $x\in\left[\frac{1}{2},1\right]$

or

Simplify the expressions : $\tan^{-1}\frac{x}{\sqrt{a^2-x^2}}$

6. Using Elementary Row Transformation, find the inverse of the matrices, if it exist, $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$

7. If $y = A\sin mx + B\cos mx$, Show that $y_2 + m^2 y = 0$

8. If $y = \sqrt{x + \sqrt{x + \sqrt{x + ...\infty}}}$ prove that $\frac{dy}{dx} = \frac{1}{2y-1}$

or

Differentiate $\sin^2 x$ with respect to $(\log x)^2$

9. Evaluate: $\int_{\pi/3}^{\pi/2} \frac{(1+\cos x)^{\frac{3}{2}}}{(1-\cos x)^{\frac{5}{2}}} dx$

or

Evaluate: $\int_{-\pi/2}^{\pi/2} \log\left(\frac{1-\tan x}{1+\tan x}\right) dx$

10. Determine the order and degree of the following differential eqations.State also, whether it is linear or non linear $9\frac{d^2y}{dx^2} = \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}$

11. If the sum of two unit vectors is a unit vector prove that the magnitude of their difference is $\sqrt{3}$.

12. For any three vectors $\vec{a}, \vec{b}, \vec{c}$ prove that $[.\vec{a}+\vec{b}, \vec{b}+\vec{c}, \vec{c}+\vec{a},]$ =2$[\vec{a}, , \vec{b}, , \vec{c},]$

SECTION -C

13. Define a binary operation * on the set {0, 1, 2, 3, 4, 5} as a * b = $\begin{cases} a+b & \text{if} \quad a+b<6 \\ a+b-6 & \text{if} \quad a+b \geq 6 \end{cases}$

Show that zero is the identity for this operation and each element a of the set is invertible with (6 – a) being the inverse of "a"

14. Consider f : R → [4, ∞] given by f(x) = x^2 + 4. Show that f is invertible with the inverse of f^{-1} of f given by f^{-1} (y) = $\sqrt{y-4}$, where R_+ is the set of all non-negative real numbers.

or

Let f : N → N defined by f(n) = $\begin{cases} \frac{n+1}{2}, & \text{if n is odd} \\ \frac{n}{2}, & \text{if n is even} \end{cases}$ for all n ∈ N. State whether the function f is bijective. Justify your answer.

15. Prove using properties of determinant: $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+1\right)$

16. Evaluate : $\int \frac{dx}{\sqrt{1-e^x}}$

17. Evaluate : $\int \frac{1}{4x^2-4x+3}dx$

or

Evaluate : $\int \frac{3\sin x + 2\cos x}{3\cos x + 2\sin x}$

18. Evaluate : $\int_0^1 \frac{\log(1+x)}{1+x^2}dx$

19. Solve the differential equation $\frac{dy}{dx} + y\tan x = 2x + x^2 \tan x$ given that y(0) = 1

20. Find the direction cosines l, m, n of two lines are connected by the relations :l + m + n = 0 , 2lm + 2ln – mn = 0, find them

or

Find the foot of the perpendicular drawn from the point (0, 2, 7) to the line $\frac{x+2}{-1} = \frac{y-1}{3} = \frac{z-3}{-2}$. Also find the length of the perpendicular.

21. A die is tossed twice. Getting a number greater than 4" is considered a success. Find the mean and variance of the probability distribution of the number of successes

22. If f(x) = $\begin{cases} \frac{x^2}{a}, & 0 \le x < 1 \\ a, & 1 \le x < \sqrt{2} \\ \frac{2b^2-4b}{x^2}, & \sqrt{2} \le x < \infty \end{cases}$ is continuous for 0 ≤ x < ∞, find a, b.

23. Water is running into an inverted cone at the rate of □ cubic meters per minute. The height of the cone is 10 meters, and the radius of its base is 5 m. How fast the water level is rising when the water stands 7.5 m above the base.

SECTION –D

24. If $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$, find A^{-1} and hence solve the system of linear equations : $x + 2y + z = 4$, $-x + y + z = 0$, $x - 3y + z = 2$.

or

If $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$, verify that $A^3 - 6A^2 + 9A - 4I = 0$ and hence find A^{-1}

25. Prove that the radius of the right circular cylinder of greatest curved surface which can be inscribed in a given cone is half of the cone.

or

A water tank has the shape of an inverted right circular cone with its axis vertical and vertex lowermost. Its semi vertical angle is $\tan^{-1}(0.5)$. Water is poured into it a constant rate of 5 cubic metre per hour. Find the rate at which the level of the water is rising at the instant when the depth of water in tank is 4m

26. Using integration find the area of the region $\{(x, y): 0 \le y \le x^2 + 1, 0 \le y \le x + 1, 0 \le x \le 2\}$

27. In a test, an examinee either guesses or copies or knows the answer to a multiple choice question with four choices. The probability that he makes a guess is $\frac{1}{3}$ and the probability that he copies the answer is $\frac{1}{6}$. The probability that his answer is correct, given that he copied it is $\frac{1}{8}$. The probability that his answer is correct given that he gussed it is $\frac{1}{4}$. What is the probability that he knows the answer to the question, given that he correctly answered it?

or

A and *B* throw a pair of dice. A wins, if he throws 7 before *B* throws 8 and *B* wins, if he throws 8 before A throws 7. If A begins, show that his chance of winning is $\frac{36}{61}$. What is *B's* Chance for winning?

28. A dietician has to develop a special diet using 2 foods P & Q each packet containing 30g of food P contains 12 units of calcium, 4 units of iron, 6 unit of cholestrol and 6 units of vitamin A, each packet of same quantity of food Q contin 3 units of calcium, 20 units of iron, 4 unit of cholestrol & 3 unit of vitamin A. The diet require atleast 240 units of calcium, 460 units of iron and at most 300 units of cholestrol. How many packets of each food should be used to minimise the amount of vitamin A in diet ? What is minimum amount of vitamin A ?

29. Find the image of the point(1, 3, 4) in the plane $2x - y + z + 3 = 0$.